Stockton University
Mathematical Mayhem 2018
Individual Round with Answers

April 14, 2018

Instructions:

This round consists of **18** problems worth a total of **80** points, made up of 8 Appetizers worth 3 points each, 7 Entrées worth 5 points each, and 3 Desserts worth 7 points each.

Each of the 18 problems is multiple choice, and each problem comes with **5** possible answers.

For each problem, **mark your answer on the answer sheet**.

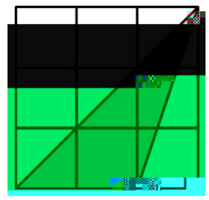
You are not required to show any work this round.

No calculators are permitted.

This round is **75 minutes** long. **Good Luck!**

| Appetizers |

Problem 1. In the diagram, each small square is 1 cm by 1 cm. What is the area of the shaded region, in square centimeters?



(A.) 2.75 (B.) 3 (C.) 3.25 (D.) 4.5 (E.) 6

Solution to Question 1. (B).

Problem 2. Which value of n makes the following identity true $a^n = [\sqrt[3]{\sqrt[6]{a^9}}]^4 [\sqrt[6]{\sqrt[3]{a^9}}]^4$?

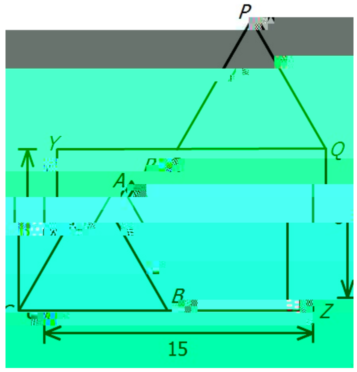
(A.) 2 (B.) 16 (C.) 8 (D.) 12 (E.) 4

Solution to Question 7. (B). When

} Entrées }

Problem 9. What is the result after $\frac{a^{4} \quad b^{4}}{a^{2} \quad b^{a}}$

tively. What is the length of AP?



(A.) 10 (B.) $\sqrt{117}$ (C.) 9 (D.) 8 (E.) $\sqrt{72}$

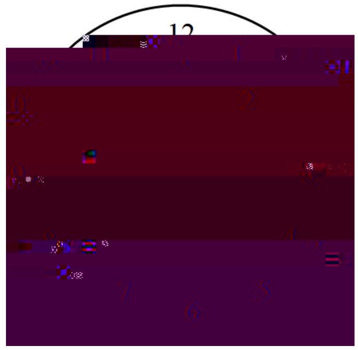
Solution to Question 14. (A). The translation that takes C to A also takes R to P. Since translation preserves distance, $AP = CR$. Since QR is length 9, RY is length 6. By the Pythaogrean Theorem, RC is length 10.

Problem 15. If $(1.0000042376)^2 = 1.00000xyz521795725376$, what is the value of $x + y + z$?
(A.) 15 (B.) 16 (C.) 17 (D.) 18 (E.) 19

Solution to Question 15. (E). Let $t = .0000042376$ and $(1+t)^2 = 1 + 2t + t^2$. Observe that t^2 is too small to affect the first three nonzero digits of $2t$, so $xyz = 847$ and $x + y + z = 19$.

~ Desserts ~

Problem 16. At some time between 9:30 and 10 o'clock the triangle determined by the minute hand and the hour hand is an isosceles triangle (see diagram). If the two equal angles in the triangle are twice as large as the third angle, what is the time?



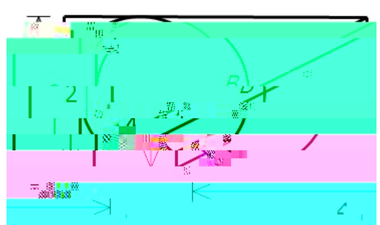
(A.) 9:39 (B.) 9:38 (C.) 9:37 (D.) 9:36 (E.) 9:35

Solution to Question 16. (D). Note that the angle made by the minute and hour hands is 72 which is one fifth of the total 360 around the clock. Let m be the number of minutes passed 9 of the time. Then $45+5m=60$ gives the position of the hour hand in minutes passed 9 and m gives the position of the minute hand. Further, $72 = 360 =5$ corresponds to 60 minutes=5 = 12 minutes. Hence

$$45 + 5m=60 = m + 12.$$

Solving for m gives $m = 33(60=55) = 36$ and the current time is 9:36.

Problem 17. A circle is tangent to three sides of a rectangle having side lengths 2 and 4 as shown. A diagonal of the rectangle intersects the circle at points A and B. What is the length of the segment AB?



(A.) $\sqrt{5}$ (B.) $\sqrt{5} \quad \frac{1}{6}$ (C.) $\sqrt{5} \quad \frac{1}{5}$ (D.) $\frac{4\sqrt{5}}{5}$ (E.) $\frac{5\sqrt{5}}{6}$

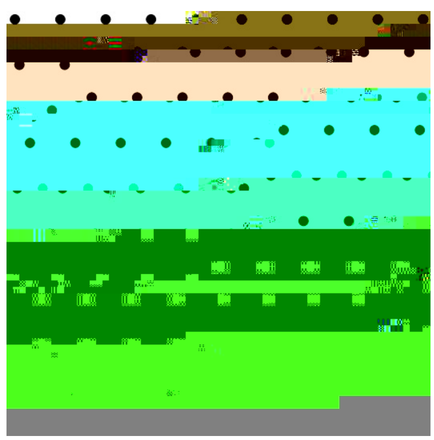
Solution to Question 17. (D). Let O be the center of the circle, C be the point of tangency between the circle and the left side of the rectangle, D be the point of tangency between the circle and the bottom of the rectangle, and E be the bottom left corner of the rectangle. Then $OCED$ is a square since the lengths of OC and OD are equal and the interior angles at C, E, and D are all right angles. So, if we can setablish a coordinate system by setting O to be the origin. Further, we can let C be $(\ 1, 0)$, E be $(\ 1, \ \ 1)$, and D be $(0, \ \ 1)$. Then the upper right corner of the rectangle is $(3, 1)$. Hence, the line segment is the line through $(\ 1, \ \ 1)$ and $(3, 1)$. This is given by the equation $y = \frac{x}{2} \quad \frac{1}{2} = \frac{1}{2}(x \quad 1)$. Further, the circle is given by the equation $x^2 + y^2 = 1$. We solve for the points of intersection A and B. Note that $y^2 = 1 - x^2 = \frac{1}{4}(x-1)^2$. Further,

$$\begin{aligned} 1 - x^2 &= \frac{1}{4}(x-1)^2 = \frac{1}{4}(x^2 - 2x + 1) = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4} \\ 0 &= \frac{5}{4}x^2 - \frac{1}{2}x - \frac{3}{4} \\ 0 &= 5x^2 - 2x - 3 \end{aligned}$$

Note that $x = \frac{2 \pm \sqrt{4-4(-3)(5)}}{10} = \frac{2 \pm 8}{10}$ and so $x = -3/5$ or $x = 1$. Solving for the corresponding y values, we get $-4/5 = \frac{1}{2}((-3/5) - 1)$ and $0 = \frac{1}{2}(1-1)$. Hence the points of intersection are $(-3/5, -4/5)$ and $(1, 0)$, so we can solve for the length AB by using the distance formula:

$$AB = \sqrt{(-3/5 - 1)^2 + (-4/5 - 0)^2} = \sqrt{64/25 + 16/25} = \frac{4\sqrt{5}}{5}.$$

Problem 18. In the square array of dots with 10 rows and 10 columns of dots shown below each dot is colored either red or blue. When two dots of the same color are adjacent in the same row or column, they are joined by a line segment that is the same color as the dots. If they are adjacent, but are of different colors, they are joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners and an additional 16 red dots that are on the edges (but not corners) of the array. The rest of the red dots are inside the array. There are 98 green segments. How many blue line segments are there?



(A.) 36 (B.) 37 (C.) 38 (D.) 39 (E.) 40

Solution to Question 18. (B). There are 9 line segments in each row and in each column, so there are 180 line segments total. Let B be the number of blue segments and R be the number of red segments. Then $B + R + 98 = 180$, and so $B + R = 82$, as there are 98 green line segments.

Coming out of a red dots, there can only be a green line segment or a red line segment. We count the total number of line segments starting from red dots. Note that in this total, the green segments are counted once and the red segments are counted twice, as the red segments have both ends at red dots. There are two edges coming from a corner dots, three edges from a non-corner edge dot, and four edges coming from an interior dot.

So, the total number of edges coming from red dots, with red edges counted twice and green edges counted once, is:

$$2(2) + 3(16) + 4(52 \quad 16 \quad 2) = 2R + 98.$$

Hence, $4 + 48 + 136 = 188 = 2R + 98$, and so $R = 45$. Finally, $B = 82 \quad 45 = 37$.